Finance Mathematics

Part 1: Terms and their meaning.

Watch the video describing call and put options at http://www.youtube.com/watch?v=EfmTWu2yn5Q and use http://www.investopedia.com or a search. Look up the following terms. Be sure your definitions apply to financial terms. Give an appropriate, related example.

Stock: *A share of a company held by an individual or group. Corporations raise capital by issuing stocks and entitle the stock owners (shareholders) to partial ownership of the corporation.*

Example: *GOOG -> google stock, PFE -> Pfizer stock*

Call Option: *An agreement that gives an investor the right (but not the obligation) to buy a stock, bond, commodity, or other instrument at a specified price within a specific time period.*

Example: *You buy a call option on shares of Google with a strike price (desired price) of $50. If the cost rises above $50 on the call date (expiration date), you will be able to purchase it the stock at $50. If the price goes down, your call option is not worth anything, since you can buy the stock for cheaper anyway.*

Put Option: *An option contract giving the owner the right, but not the obligation, to sell a specified amount of an underlying security at a specified price within a specified time.*

Example: *You buy a put option on shares of Intel with a strike price of $40. If the price goes below $40 on the expiration date, you will be able to sell the stock at $40 and make money. Otherwise, if the price rises, you lose the money you spent on the option, since you can sell it for more on the market.*

Volatility of a stock: *A statistical measure of the dispersion of returns for a given security or market index. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. Commonly, the higher the volatility, the riskier the security.*

Risk Free Interest Rate: *The theoretical rate of return of an investment with zero risk. The risk-free rate represents the interest an investor would expect from an absolutely risk-free investment over a specified period of time.*

Common approximation*: the interest rate on a three-month U.S. Treasury bill is often used as the risk-free rate.*

Compounding: *Interest calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan. Compound interest can be thought of as "interest on interest," and will make a deposit or loan grow at a faster rate than simple interest, which is interest calculated only on the principal amount. The rate at which compound interest accrues depends on the frequency of compounding; the higher the number of compounding periods, the greater the compound interest*.

Compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

Continuous compound interest formula: $A = Pe^{rt}$

Part 2: Create an Excel workbook to calculate:

- Future value of an investment compounded n times per year at r% interest for t years.
- Same as above, but compounded continuously
- Graph each of the above versus time.

Use the Excel workbook you created to find the following:

a) The value after 4 years, of a savings account of $500 at an interest rate of 2.1% compounded monthly.

$543.77

b) The same account with $250 at an interest rate of 4.2% compounded monthly.

$295.65

   a. Explain the similarities and differences between the two parts above. Do they make intuitive sense? Why or why not? *Even though the amount halved and interest doubled, the calculation wasn't equal because you are taking a larger percent, albeit of a smaller amount, resulting in a smaller balance.*

c) The value of a savings account of $500 at an interest rate of 2.1% compounded weekly.

$295.71

d) The same as above, except compounded continuously.

$295.73

   a. Explain the similarities and differences between parts a, c, and d. Do your results make sense? Why or why not? *The continuous compounding is adding a smaller percentage, but doing it all the time. The cumulative effect amounts to more total, although not very much more, at least for a short time.*

Suppose you wanted to have at least $1,000 by the time you graduated college, and assume the interest rate remains at 2.1%, compounded monthly, throughout the 4 years you are in school. Create an Excel worksheet to calculate the amount you'd have to start with, as well as the following:

$919.50

a) The same situation, except the interest rate is 3.1%.

$883.52

b) Suppose you want to have the same amount you'd get in the scenario in part a in the previous question, except you need the money in only 3 years. How much would you have to invest?

$911.30

Part 3: Use the Financial Math Excel workbook (and the Other Values tab) to find the value of an option.

a) Suppose you feel a stock will go up in value, and you want to hold the cost of a stock with the option of purchasing later. Find the cost of a call option with a strike price of $110 on a stock priced at $110 over a maturity period of 3 years, assuming a risk-free interest rate of 3.5% and a market volatility of 20%.

$20.50

b) Suppose instead you expect the stock to go down in value, and want to sell the stock at a later time at its current value. Use the parameters above to do the same calculation for the cost of a **put option**.

$9.54

c) Which is greater (call or put)? Try changing the values of the stock price, strike price, time period, risk-free interest, and market volatility and compare the call and put option prices for each set of values. Is the one always greater? *When the strike price is "enough greater than" the stock price, the put price is less than the call price. Otherwise, the put option price will be less. This is due to the nature of an expected rising market.*

d) Suppose the risk-free interest rate were 5% instead of 3.5%. Calculate the percent change in the call price for these conditions. Is it more or less than 1.5%? *$23.02; the change is about 12.3%, definitely not linear*

e) Look at the graph "Rho: ΔOption Cost/ΔRisk Free Rate"under the tab "Other Graphs". This graph measures the rate of change between these two variables. If you enter the initial data and observe the graph, what does the shape tell you about the cost of the call option for interest rates between 0 and 20%? What happens to the cost of the call option when the interest rate rises above about 20%? Remember, this graph shows the rate of change, not the actual values. *Cost is increasing more and more rapidly as the interest rate approaches 20%, beyond 20% it continues to grow, but the rate of growth decreases*

f) Now look at the graph labeled "Vega: …" on the same tab. Notice the 20% factor in this graph, as well. Volatility measures the random, unpredictable nature of the market. Describe the market for options in terms of the volatility. *The cost of the option grows very rapidly for values of volatility less than 20%. The cost of the option would grow less rapidly for increases in volatility started off greater than 20%.*

g) Look at the graphs under the tab "Option vs Time". What do these graphs tell you as an investor in call options or as an investor of put options? Explain. *The graphs of option vs. time and theta are almost flips of one another. Put options costs tend to decrease over time, whereas call options tend to increase. As an investor, I see that the shorter the time of contract, the more likely the price of the option will rise (or fall, for a put option) rapidly. Over very long time periods, the cost of the option changes at a steadier rate.*

h) Turn to the tab labelled "Option vs. Stock". The graphs show the cost of a share of an option versus the cost of the stock, the rate of change, and the rate of change of the rate of change. Why do the graphs have such different shapes? *It appears from the first graph that the option price nearly grows linearly as the stock price increases. However, Delta (graph 2) shows that it grows rapidly for smaller values of initial stock cost, and then grows less rapidly for larger values of stock. Gamma has a peak, showing the values of stock cost where the cost of the option grows most rapidly. The graphs work with the fundamental data, the rate of change, and the rate of acceleration of change of the option cost for given values of stock cost.*

i) Gamma seems to have a peak. Look at the other two graphs for the same x value (cost of stock). What is going on with the costs at this particular value? *Explained above. The option cost is growing most rapidly for stock prices at that peak.*

a) Start with the original values given in part a, above. Change each of the variables (stock price won't affect the graphs, since it's the independent variables) by the same amount (say, each +10% and -10%). Which change has the biggest effect on option cost? As an investor, what would you be most concerned about in order to ensure the highest, and safest return? [Fill in the table on the next page, then use the results to answer the question. Note: +5% means 100% + 5% = 105% of the original, i.e., multiply the original by 1.05, other increases or decreases are defined similarly.]

	Stock cost	Strike price	Risk free rate	Volatility	Contract time	Option Cost
Original value	110	110	3.5%	20%	3	20.50
Stock + 5%	*115.50*	*110*	*3.5%*	*20%*	*3*	24.50
Stock + 10%	*121*	*110*	*3.5%*	*20%*	*3*	28.54
Stock - 5%	*104.50*	*110*	*3.5%*	*20%*	*3*	16.89
Stock – 10%	*99*	*110*	*3.5%*	*20%*	*3*	13.60
Strike + 5%	*110*	*115.50*	*3.5%*	*20%*	*3*	17.91
Strike + 10%	*110*	*121*	*3.5%*	*20%*	*3*	15.59
Strike - 5%	*110*	*104.50*	*3.5%*	*20%*	*3*	23.38
Strike – 10%	*110*	*99*	*3.5%*	*20%*	*3*	26.54
rate + 5%	*110*	*110*	*3.675%*	*20%*	*3*	20.79
rate + 10%	*110*	*110*	*3.85%*	*20%*	*3*	21.08
rate - 5%	*110*	*110*	*3.325%*	*20%*	*3*	20.22
rate – 10%	*110*	*110*	*3.15%*	*20%*	*3*	19.93
vol + 5%	*110*	*110*	*3.5%*	*21%*	*3*	21.18
vol +10%	*110*	*110*	*3.5%*	*22%*	*3*	21.86
vol - 5%	*110*	*110*	*3.5%*	*19%*	*3*	19.82
vol – 10%	*110*	*110*	*3.5%*	*18%*	*3*	19.15
time + 5%	*110*	*110*	*3.5%*	*20%*	*3.675*	23.22
time + 10%	*110*	*110*	*3.5%*	*20%*	*3.85*	23.89
time - 5%	*110*	*110*	*3.5%*	*20%*	*3.325*	21.83
time – 10%	*110*	*110*	*3.5%*	*20%*	*3.15*	21.12

Note that ±5% does not mean add 5% to the percentage. Note that the cost of the call option is directly proportional to all of the variables except for strike price. Time and stock price had the biggest influences on option cost, reminiscent of the compounding formulas. Considering where we start, we can look at the graphs to

get an idea of how the option cost changes with corresponding changes in other variables. The changes in any initial values do change the graphs, though. Play around to see how widely they vary.