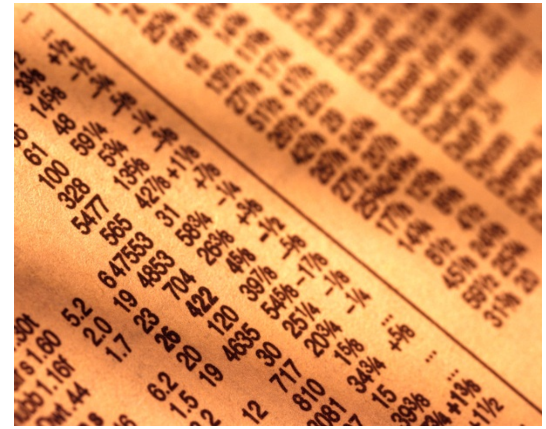


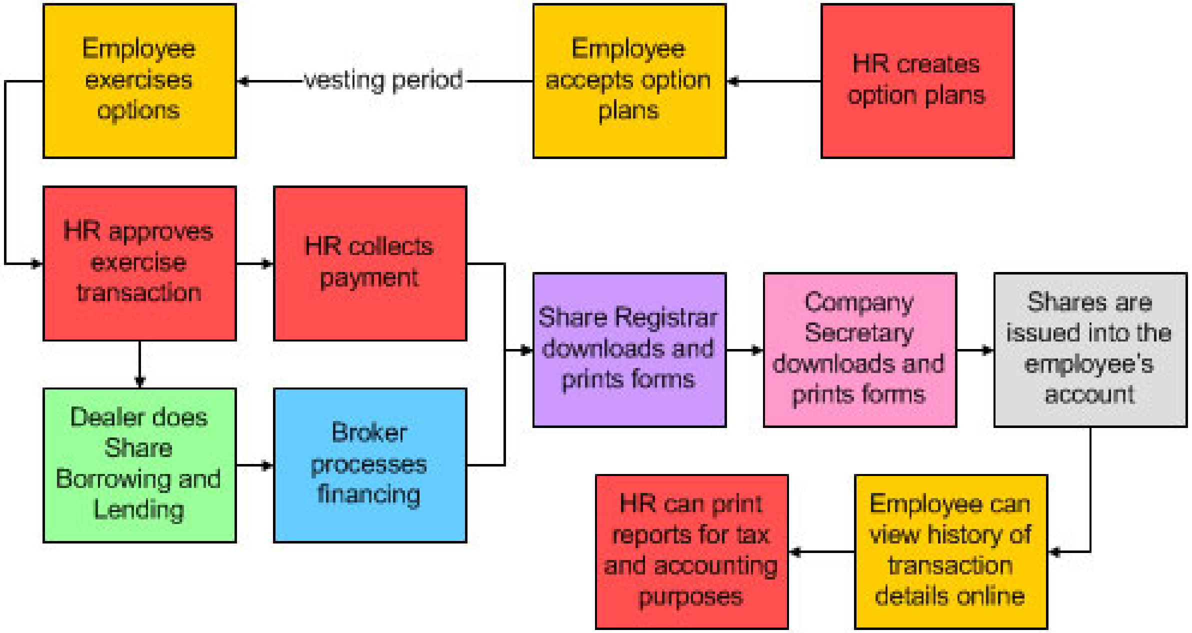
Mathematical Finance
Why would you need to know what an option is?

Presenters

Ron Grosz

Patricia Parker-Davis

Employee Stock Options



Some companies offer employees special stock options depending upon employee performance.

European Options

There are two types of European Options

- European Call Option (ECO)- a financial derivative that gives the owner the option, **but not the obligation** to **buy** an asset at a predetermined **strike price, which is called K, at the maturity date**.

- European Put Option(EPO) is a financial derivative which gives the owner the option, **but not the obligation**, to **sell** an asset at a predetermined strike price, K, at the predetermined maturity date.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- Let's look at some of the terms used:

- **Strike price**-A strike price, also referred to as an execution price, represents the price at which a securities contract may be exercised, that is either bought or sold. It is most common in options trading. **Options** are derivative contracts that provide investors with the "option" to purchase or sell an asset, such as a stock, when it reaches a strike price. When used effectively, an options strike price can significantly enhance an investors holdings, but under certain conditions may also seriously hurt returns (WiseGEEK.com)

- **Maturity date**-Maturity date refers to the date on which an issuer of a credit obligation, or bond, must pay back the principal amount of the debt to those who purchased it. Referred to as **T for Time**
- The expiration date for all listed stock options in the United States is normally the third Friday of the contract month, which is the month when the contract expires.

- **Rate -** Interest rate options from exchanges in the United States are offered on Treasury bond futures, Treasury note futures and eurodollar futures.
- **Dividend -** A distribution of a portion of a company's earnings
- **Volatility call σ -** A variable in option pricing formulas showing the extent to which the return of the underlying asset will fluctuate between now and the option's expiration. Volatility, as expressed as a percentage coefficient within option-pricing formulas, arises from daily trading activities. How volatility is measured will affect the value of the coefficient used.

In options trading, **an option is out of the money** when it has no financial value, making it worthless to the person who holds it because of a shift in market movements. Exercising the option would result in a loss, and the holder may simply allow it to expire without taking any action. This state is the opposite of **being in the money**, where the contract has value, making it appealing to exercise. Options traders use a variety of techniques to avoid situations where their options lose value and end up out of the money.

The Black-Schole formula

Black–Scholes–Merton model is a mathematical model of a financial market containing certain derivative investment instruments. From the model, one can deduce the **Black–Scholes formula**, which gives a theoretical estimate of the price of European-style options.

The Black–Scholes model was first published by Fischer Black and Myron Scholes in their 1973 paper, "The Pricing of Options and Corporate Liabilities“.

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Lets find an European Call Option with a Dividend based on Pepsico for July 31 2014

"y" is an amount to adjust for Divided paid on the stock.

$$y = \ln\left[\frac{S_o}{S_o - De^{-r}}\right]$$

δ

Equation of the European Call Option with a Dividend

With starting price of S_o ,Strike Price of K, Time of (T), adjustment of (y) and rate (r).

$$V^{y} = e^{-yT} S_o N(d_{+}) - e^{-rT} K N(d_{-})$$

Define N(d_+)

Finding d_+; using some calculus to find N(d_+)

$$d_+ = \frac{\ln\left(\frac{S_o}{K}\right) + \left(r - y + \sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(d_+) = \int_{-\infty}^{d_+} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\,dx$$

Define N(d_-)

The same way to find N(d_-)

$$d_- = \frac{\ln\left(\frac{S_o}{K}\right) + \left(r - y - \sigma^2\right)T}{\sigma\sqrt{T}}$$

$$N(d_+) = \int_{-\infty}^{d_-} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\,dx$$

Finding y

If S_o = 89.20, D = .655 and r = .03%, then

$$y = ln\left[\frac{89.2}{89.2 - .655^{-.0003*.25}}\right] \approx 0.011$$

Find d_+ and d_-

$$d_+ = \frac{ln\left(\frac{89.2}{90}\right) + (0.0003 - 0.011 - .06^2/2).25}{.06\sqrt{.25}} \approx .361$$

$$d_- = \frac{\ln\left(\frac{89.2}{90}\right) + \left(.0003 - .011 - .06^2 / 2\right).25}{.06\sqrt{.25}} \approx 0.338$$

Find $N(d_+)$ and $N(d_-)$

$$N(d_+) = \int_{-\infty}^{-0.356} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\,dx \approx 0.361$$

$$N(d_+) = \int_{-\infty}^{-0.416} \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}}\,dx \approx 0.338$$

δ

Finding the European Call Option with a Dividend using the equation

S_o =90.14 , K =91 , T = .25 years,

r = .03%, $\boldsymbol{\sigma} = 6\%$,

$N(d_+) = 0.361$, $N(d_-) = 0.338$, $y = .011$

$$V^y = e^{-yT} S_o N(d_+) - e^{-rT} K N(d_-)$$

δ

Finding the European Call Option with a Dividend

S_o =90.14 , K =91 , T = .25 years,

r = .03%, $\boldsymbol{\sigma} = 6\%$,

$N(d_+) = 0.361$, $N(d_-) = 0.338$, $y = .011$

$$V^y = e^{-yT} S_o N(d_+) - e^{-rT} K N(d_-)$$

$$V^y = e^{-0.0003*.25}(89.2)(0.361) - e^{-0.0003*.25}(90)(0.338) \approx 1.78$$

On July 31 2014 at 10:30am

October 2014 Call option was $1.76

With not knowing the $\boldsymbol{\sigma}$ for sure.

Black-Scholes model is close to the asking price for the call option

A helpful website to write an excel way of find the price can be found at http://www.macroption.com/black-scholes-excel/ for a secure payment on a Credit Card or Paypal.

If you want to see this number in action

One of the website you can use to find an option price

http://finance.yahoo.com/q/op?s=PEP&m=2014-10

It is nice to know where the numbers come from using the Black-Schole formula

References

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