



# Free Range Voting

# Plurality Voting

- Sample plurality ballot:

Vote for one candidate

- Truth**
- Beauty**

# Plurality Voting

- Sample plurality ballot:

Vote for four candidates

**Truth**     **Strange**     **Up**  
 **Beauty**     **Charm**     **Down**

# Positional Voting

- Sample positional ballot:

Rank all the candidates

⑤ **Truth**    ④ **Strange**    ② **Up**  
① **Beauty**    ③ **Charm**    ⑥ **Down**

# Positional Voting

- A voting vector must be selected to weight each of the places
- Borda Count weights linearly from  $1^{\text{st}} = 1$  point to  $n^{\text{th}} = 0$  points
- The “vote for two of the six” situation would have the vector  $1^{\text{st}} = 2^{\text{nd}} = 1$  point,  $3^{\text{rd}} \dots 6^{\text{th}} = 0$  points

# Approval Voting

- Sample approval ballot:

Vote for any number

- Truth**     **Strange**     **Up**  
 **Beauty**     **Charm**     **Down**

# Cumulative Voting

- Sample cumulative ballot:

Cast four votes

Truth     Strange     Up  
 Beauty     Charm     Down

# Classifying the Methods

- Voting methods can be classified using two quantities:
  - quota (total votes that can be cast)
  - weight (distribution of voting vector)
- Both quantities are usually fixed
  - For example, a Borda Count of a three-candidate election requires the voter to cast two votes with a vector of  $(1, \frac{1}{2}, 0)$ .



# Degrees of Freedom

	Quota	Fixed	Variable
Weight		Fixed	Variable
		Fixed	Variable
		Variable	Variable

# Maximal Freedom?

- Allows for a variable total number of votes, i.e. a voter must be able to cast as many votes as there are candidates
- Allows for a variable voting vector, i.e. a voter must be able to allocate whatever weight he or she wishes per candidate

# Range Voting

- Sample range ballot:

Rate any or all candidates

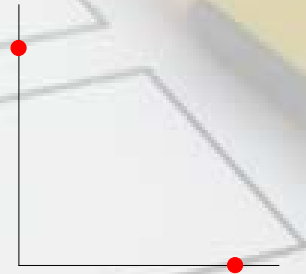
**Truth**     **2** **Strange**     **7** **Up**

**7** **Beauty**     **10** **Charm**     **0** **Down**

# Geometry of Voting

- Individual vector

- $(T,B) = (0,1)$  or  $(1,0)$
- Graph of possible vectors



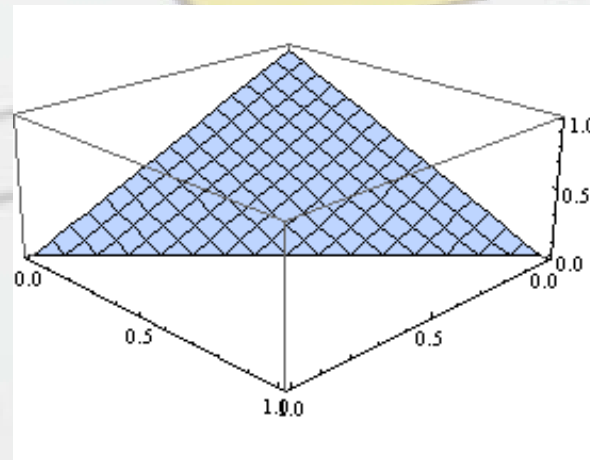
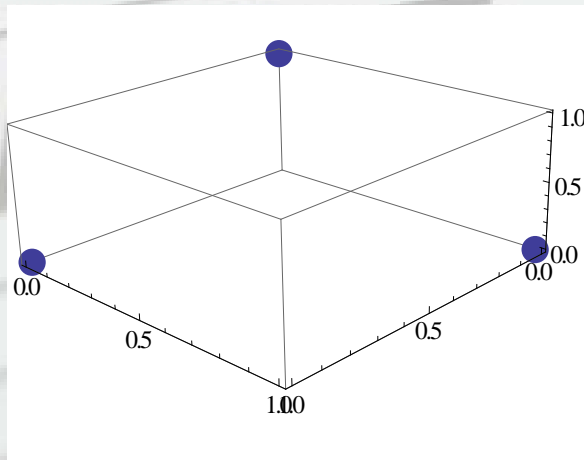
- Societal vector

- $(T,B) = (x,1-x)$ , with  $0 \leq x \leq 1$
- Graph of "hull" of possible vectors



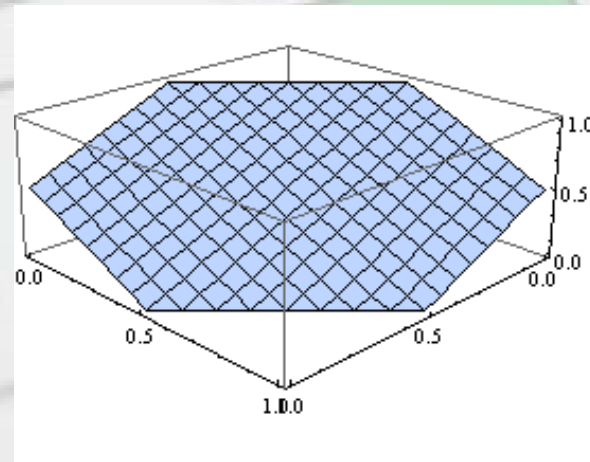
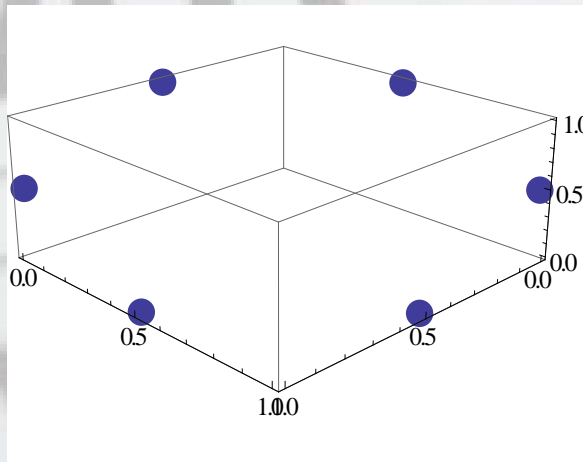
# Geometry of Voting

- For a three-candidate election...
  - Individual vectors  $(1,0,0)$ ,  $(0,1,0)$ , and  $(0,0,1)$
  - Societal vector  $(x,y,1-x-y)$ , with  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$



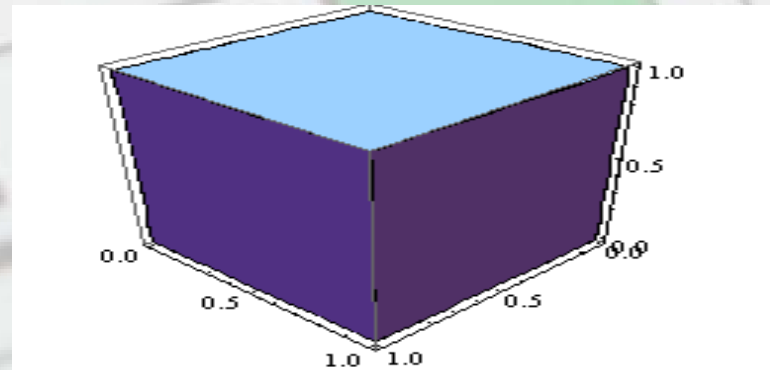
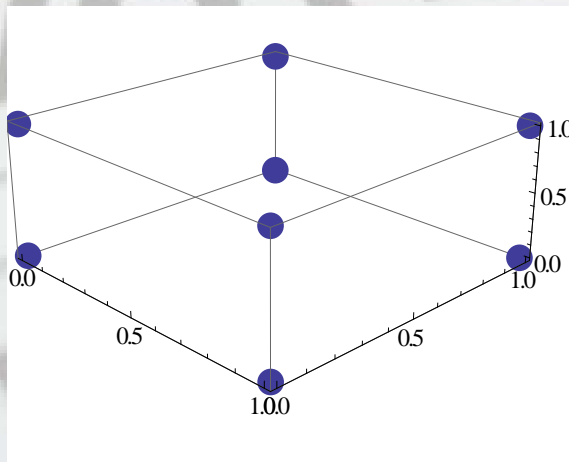
# Geometry of Voting

- For a Borda Count election...
  - Individual vectors  $(1, \frac{1}{2}, 0)$ ,  $(1, 0, \frac{1}{2})$ ,  $(\frac{1}{2}, 1, 0)$ ,  $(\frac{1}{2}, 0, 1)$ ,  $(0, 1, \frac{1}{2})$ ,  $(0, \frac{1}{2}, 1)$
  - Societal vector  $(x, y, 1.5 - x - y)$ , with  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$  [can be projected back onto  $z = 1 - x - y$ ]



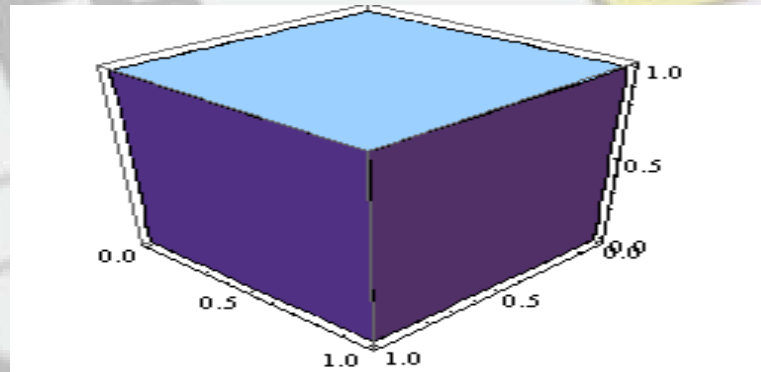
# Approval Geometry

- For an approval voting election...
  - Individual vectors  $(1,0,0)$ ,  $(1,1,0)$ ,  $(1,0,1)$ ,  $(1,1,1)$ ,  $(0,1,1)$ ,  $(0,1,0)$ ,  $(0,0,1)$ ,  $(0,0,0)$
  - Societal vector  $(x,y,z)$ , with  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ ,  $0 \leq z \leq 1$



# Range Geometry

- For a range voting election...
  - Individual vectors and societal vector are structured the same:  $(x, y, z)$ , with  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ ,  $0 \leq z \leq 1$





# Independence

- Independence of Irrelevant Alternatives (IIA)
  - One of the main criteria for a voting system to be considered “fair”
  - Candidates’ ratings are independent of each other, e.g. when one candidate withdraws the others are unaffected

# Independence

- Positional voting

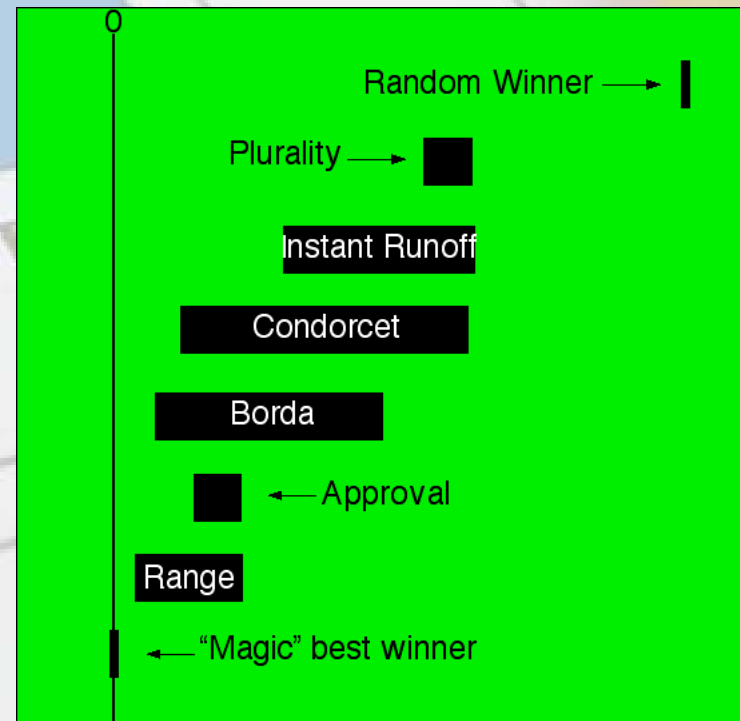
- Because  $z = W - x - y$ , the coordinates are not independent
- Cannot eliminate a coordinate without moving off the “hull” or adjusting the other coordinates, possibly changing the winner

- Range voting

- $z \neq W - x - y$ , the three coordinates are independent
- Can eliminate a coordinate by projecting down a dimension without affecting the others

# Bayesian Regret

- A mathematical measure based on the “expected avoidable human unhappiness” caused by the voting method
- Range Voting scores the best



Bayesian Regrets for several elections methods. Underlying data from Smith 2000, graphical presentation from p.239 of W.Poundstone's book *Gaming The Vote* (Hill & Wang 2008). Horizontal axis has arbitrary units; smaller regrets are better. Bars range from total voter honesty (left end) to total voter strategy (at right).

# A New System?

- Olympic judging



- Internet rating systems

**The Dark Knight (2008)**

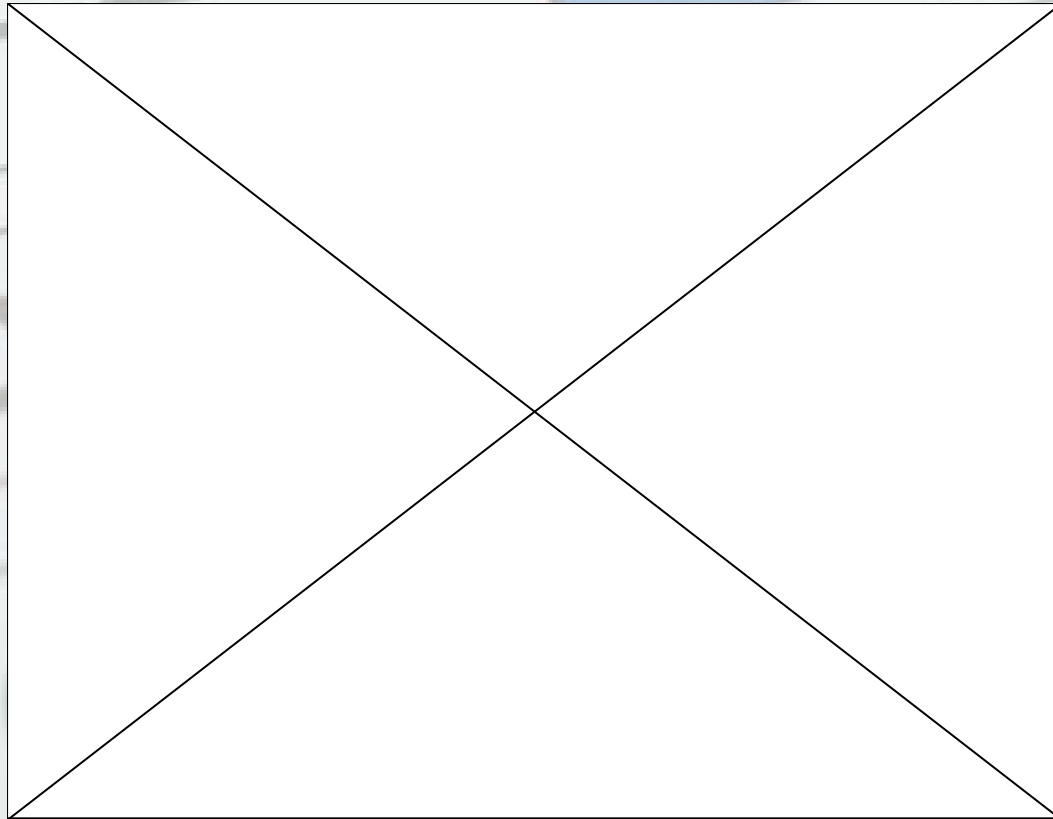
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User Rating: 9.3/10 (203,494 votes)

# A New System?

- Honeybee hive relocation



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