
Research Experience for Teachers 2007

Mathematics – Elements of Number Theory



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Remember the Remainder



A New Look at the Division Algorithm

What does NUMBER THEORY have to do with me?

- In our RET we looked at a broad range of topics in Number Theory
- One of the topics we discussed in depth was the division algorithm

The Division Algorithm

- The division algorithm is the process of writing division problems as multiplication and addition

- The division algorithm reads

$$b = n \cdot a + r$$

- As an example lets look at 7 divided by 2

$$7 = 3 \cdot 2 + 1$$

What does the Division Algorithm have to do with anything?

- In our activity we are attempting to relate the remainder of the division algorithm to writing improper fractions as mixed numbers and decimals
- While this seems complex it is actually a simple process

So what did we do anyway?

- First we looked at how many times 3 completely goes into 9, 10, and 11
- 3 divides 9, 10, and 11 completely 3 times
- However we know that $9 \div 3$, $10 \div 3$, and $11 \div 3$ are not equal to each other, so we must look at the remainder
- $9 \div 3 = 3$
 $10 \div 3 = 3$ remainder 1
 $11 \div 3 = 3$ remainder 2

Oh, the division algorithm!

- Then rewrite the equations in the form of the division algorithm
- $9 \div 3 = 3 \longrightarrow 9 = 3 \cdot 3 + 0$
- $10 \div 3 = 3 \text{ remainder } 1 \longrightarrow 10 = 3 \cdot 3 + 1$
- $11 \div 3 = 3 \text{ remainder } 2 \longrightarrow 11 = 3 \cdot 3 + 2$

How does the Division Algorithm lead to fractions?

$$9 = 3 \cdot 3 + 0$$

$$\frac{9}{3} = \frac{3 \cdot 3 + 0}{3}$$

$$\frac{9}{3} = \frac{3 \cdot 3}{3} + \frac{0}{3}$$

$$\frac{9}{3} = 3 + 0$$

$$10 = 3 \cdot 3 + 1$$

$$\frac{10}{3} = \frac{3 \cdot 3 + 1}{3}$$

$$\frac{10}{3} = \frac{3 \cdot 3}{3} + \frac{1}{3}$$

$$\frac{10}{3} = 3 + \frac{1}{3}$$

$$11 = 3 \cdot 3 + 2$$

$$\frac{11}{3} = \frac{3 \cdot 3 + 2}{3}$$

$$\frac{11}{3} = \frac{3 \cdot 3}{3} + \frac{2}{3}$$

$$\frac{11}{3} = 3 + \frac{2}{3}$$

Simplifying what we know

$$\frac{9}{3} = 3 + 0$$

$$\frac{10}{3} = 3 + \frac{1}{3}$$

$$\frac{11}{3} = 3 + \frac{2}{3}$$

- The final step of this portion is to combine our whole number and fraction parts

$$\frac{9}{3} = 3$$

$$\frac{10}{3} = 3 \frac{1}{3}$$

$$\frac{11}{3} = 3 \frac{2}{3}$$

- We have finally computed how an improper fraction changes to a mixed number using the Division Algorithm

Really the calculator?

- Next we will use the calculator to do some experimenting with the numbers we have calculated
- The calculator is an important tool and students need to learn to **use it properly**
- Students will take the improper fraction and find its value by typing it into the calculator as well as the fractional part of the mixed number by typing it into the calculator and try to relate the two parts

The Beginning of the End

$$\frac{9}{3} = 3$$

$$\frac{10}{3} = 3 \frac{1}{3}$$

$$\frac{11}{3} = 3 \frac{2}{3}$$

$$\frac{9}{3} = 3$$

$$\frac{10}{3} = \underline{3.3333\dots}$$

$$\frac{11}{3} = \underline{3.6666\dots}$$

$$\frac{9}{3} = 3 + 0$$

$$\frac{1}{3} = \mathbf{0.3333\dots}$$

$$\frac{2}{3} = \mathbf{0.6666\dots}$$

$$3 = 3 + 0$$

$$\frac{10}{3} = 3 + \frac{1}{3}$$

$$\frac{11}{3} = 3 + \frac{2}{3}$$

$$\underline{3.33\dots} = 3 + \mathbf{0.33\dots}$$

$$\underline{3.66\dots} = 3 + \mathbf{0.66\dots}$$

What is the hopeful conclusion

- In this case remember we were looking at the fraction of thirds
- Remainder 0
- The mixed number will have no fractional part
- The real number has no decimal part
- Remainder 1
- The mixed number will have a fraction of $\frac{1}{3}$
- The real number has the decimal 0.3333....
- Remainder 2
- The mixed number will have a fraction of $\frac{2}{3}$
- The real number has the decimal 0.6666....

Is that the end?

- I will use it in my Basic Skills classes, and potentially my other classes
- Other teachers in the RET are also going to try and use it with their freshman classes
- I have a 6th grade teacher who is going to try to use it in her class this year