

Bicycle Gear Simulator

by Tom Falcone and Ivan Moore



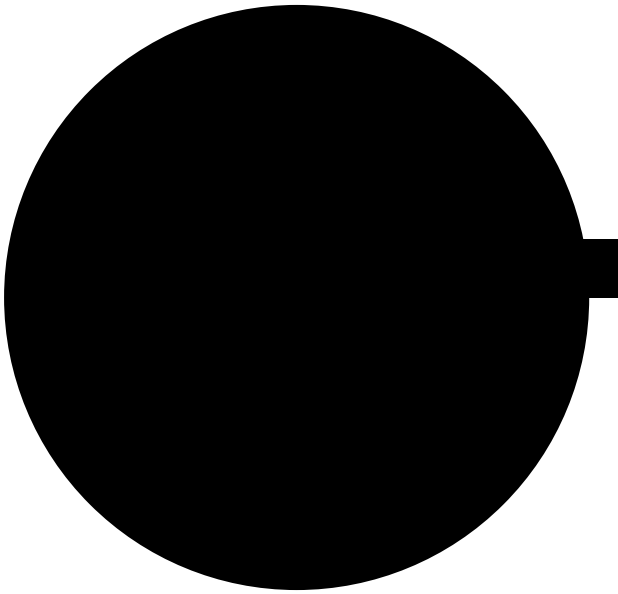
Goal and Objective:

To make an interactive tool to help students investigate and gain deeper understanding of fractions and ratios as found in a common setting.

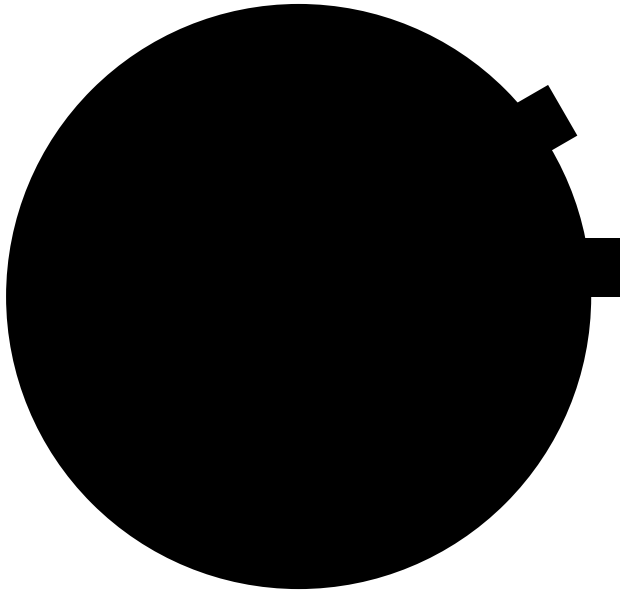


the combinations of gear ratios and wheel diameters to determine how various settings affect the distance travelled and how the gears act as levers to convert the energy applied at the pedals to motion will be an exploratory tool and an aid for students to check their own calculations.

Making the Gear
Phase 1: One tooth



Making the Gear
Phase 2: Rotating the tooth

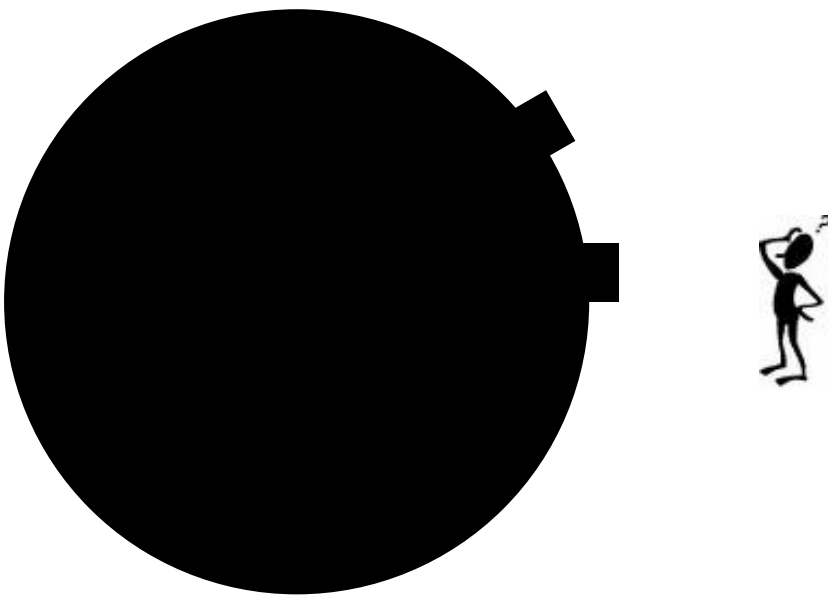


Making the Gear

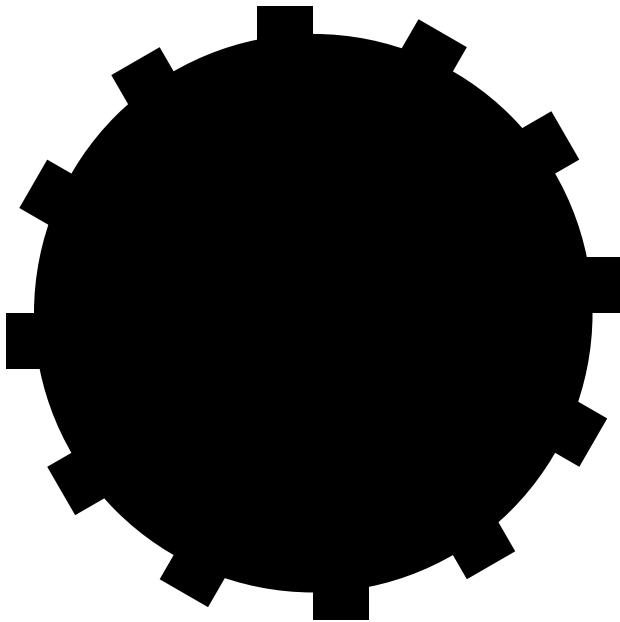
Phase 2b: Simplifying

```
myRectangle = Rectangle[{0.9, 0}, {1.1, 0.2}];

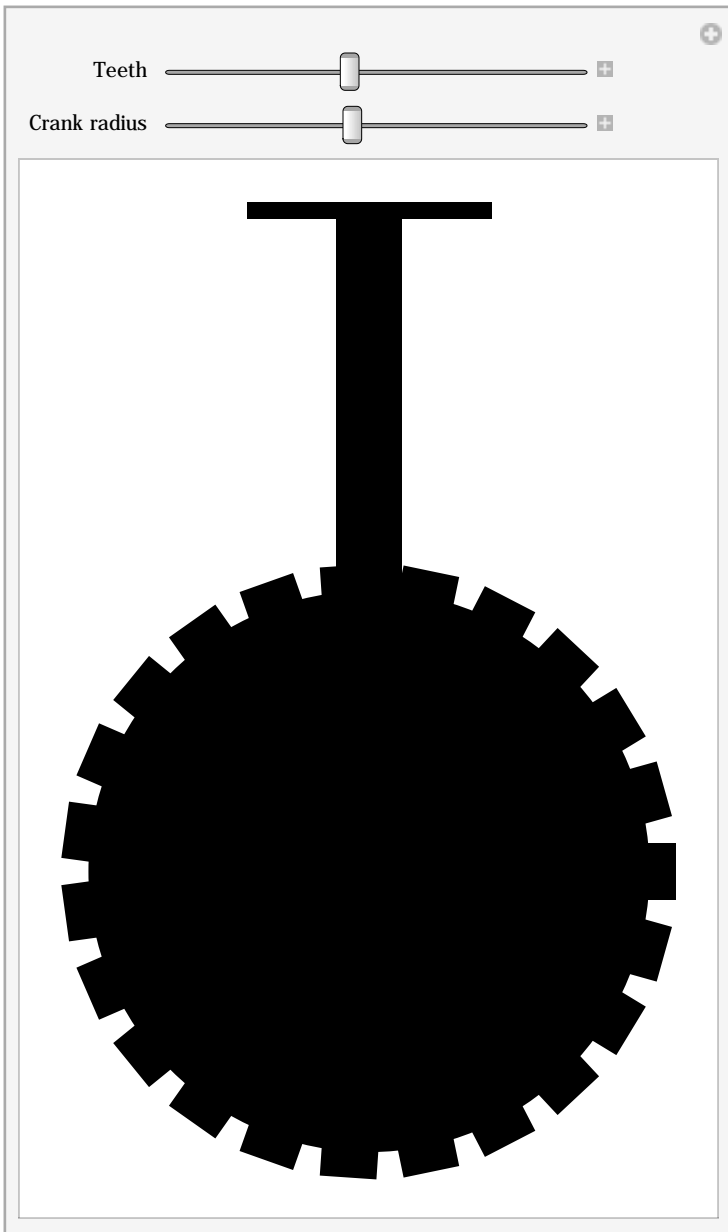
Graphics[
  {Disk[{0, 0}, 1], (* makes a solid disk centered at (0,0) with radius 1 *)
  myRectangle, (* makes a rectangle with lower right corner at (0.9,0) and upper right corner at
  (1.1, 0.2) *)
  Rotate[myRectangle, 2  $\pi$ /12, {0, 0}] (* rotates the rectangle 2  $\pi$ /12 radians about the center (0,0)
  *)
  }]
```



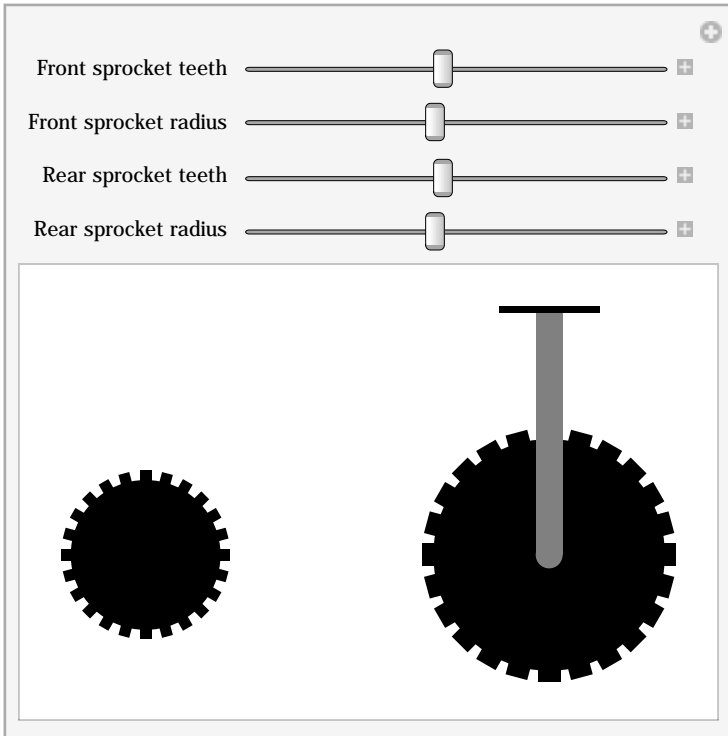
Making the Gear
Phase 3: Filling in the Teeth

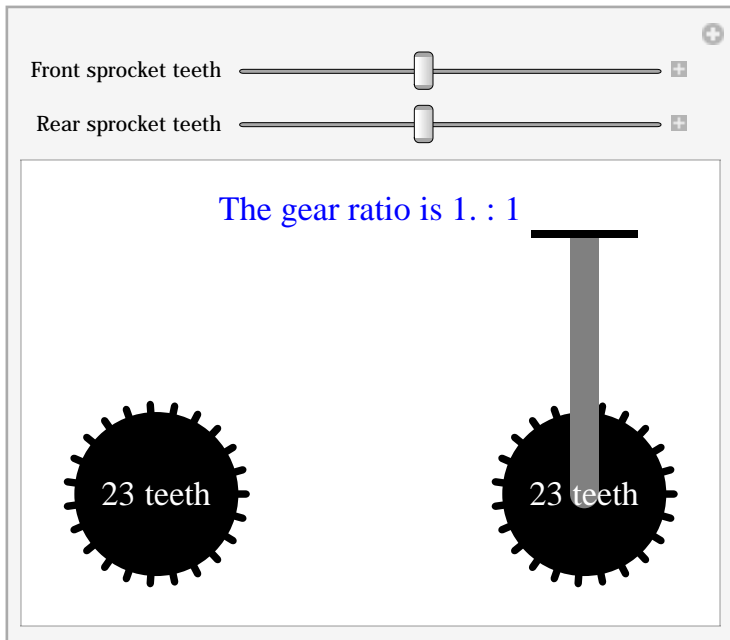


Add a Crank, Pedal, and Manipulate Controls

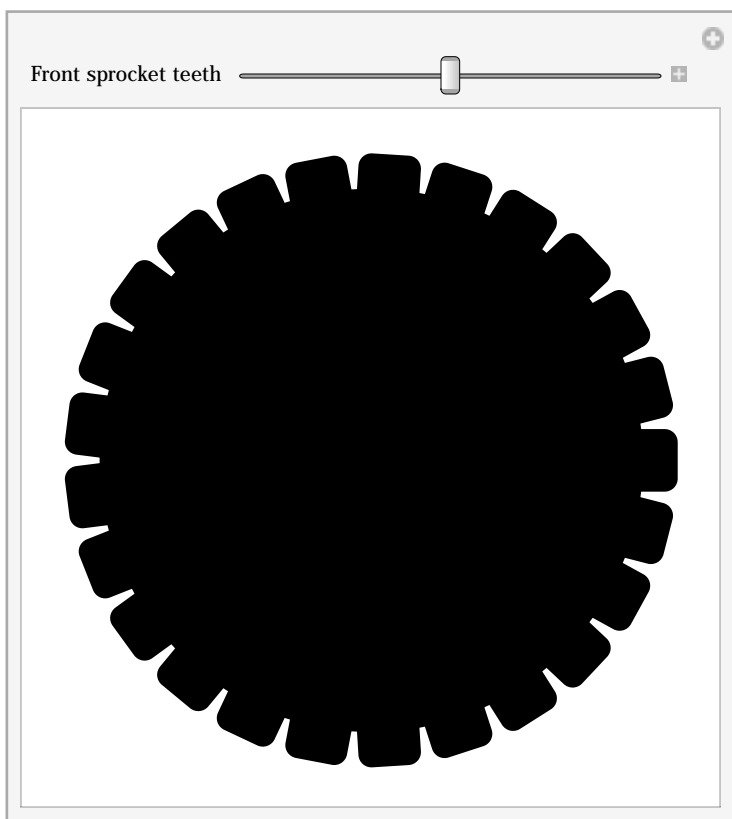


Add a Second Sprocket with Controls



Gear Radius is a Function of Number of Teeth

Analyzing the Gear Tooth Width Problem



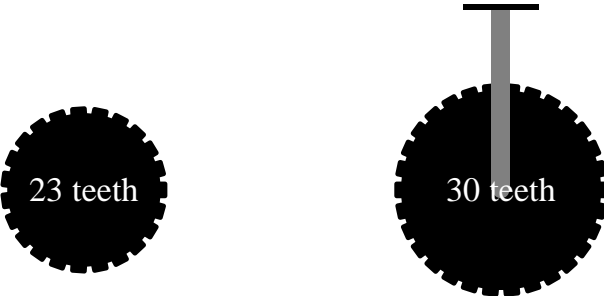
Teeth are Fixed / Controls and Text Added

Bicycle Gear Analysis

Front sprocket teeth Other # of teeth:

Rear sprocket teeth 23 Rear wheel Diameter

The gear ratio is 0.766667 : 1
Meters of development: 2.70614 meters



The diagram illustrates the gear setup. On the left is a smaller black sprocket with 23 teeth. On the right is a larger black sprocket with 30 teeth. A vertical gray line connects the two sprockets, representing the chain drive.

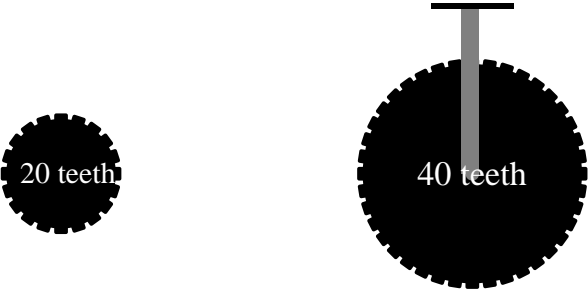
Optimizing Control Layout
Fixed the Crank Size Problem

Bicycle Gear Analysis

Front sprocket teeth Other: Rear wheel Diameter

Rear sprocket teeth 20

The gear ratio is 0.5 : 1
Meters of development: 4.14942 meters



The diagram illustrates the gear setup. On the left is a smaller black sprocket with 20 teeth. On the right is a larger black sprocket with 40 teeth, which is connected to a vertical crank arm. The text above the sprockets states the gear ratio is 0.5 : 1 and the meters of development is 4.14942 meters.

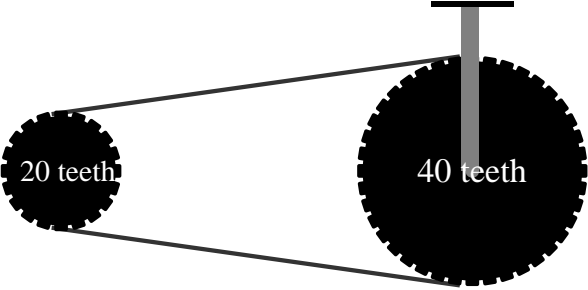
Adding a Chain

Bicycle Gear Analysis

Front sprocket teeth Other: Rear wheel Diameter

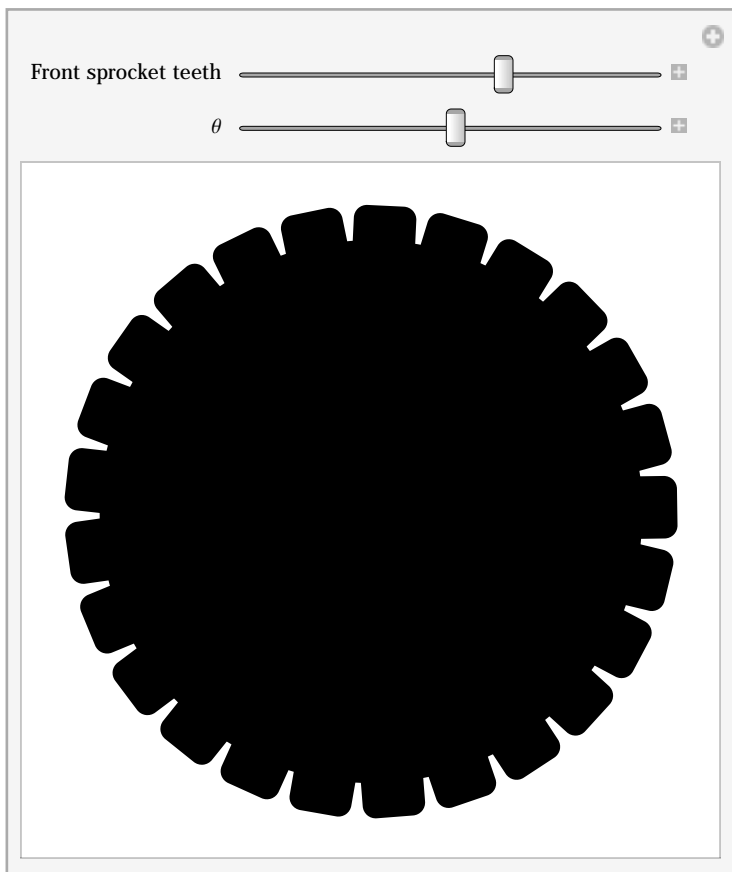
Rear sprocket teeth 20

The gear ratio is 0.5 : 1
Meters of development: 4.62819 meters



The diagram illustrates a bicycle chain drive. On the left is a smaller sprocket labeled "20 teeth". On the right is a larger sprocket labeled "40 teeth". A chain is shown connecting the two sprockets. The larger sprocket is mounted on a vertical axle with a horizontal top bar.

Animating Gears Phase 1: Rotate One Gear



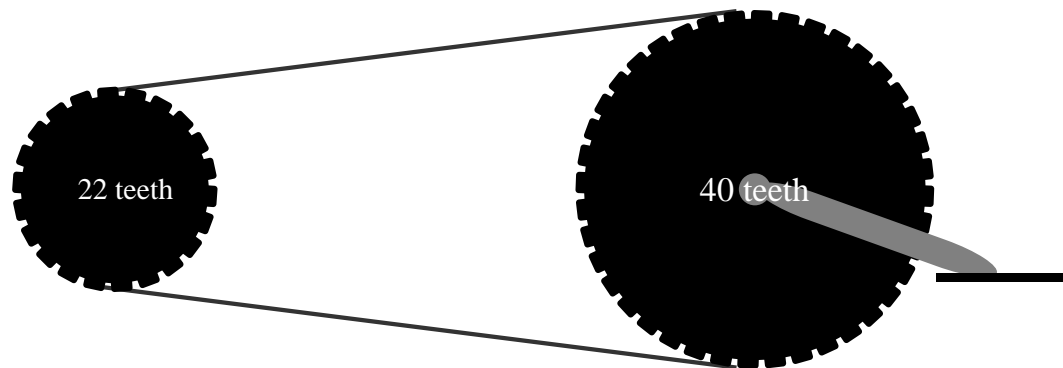
Animation

Phase 2: Gears and Pedals Rotate

Bicycle Gear Analysis

Front sprocket teeth 30 40 51 Other: Rear wheel Diameter Rear sprocket teeth 22Rotation Angle 1.916

The gear ratio is 0.55 : 1
Meters of development: 3.7722 meters



Adding a Second Pedal

Bicycle Gear Analysis

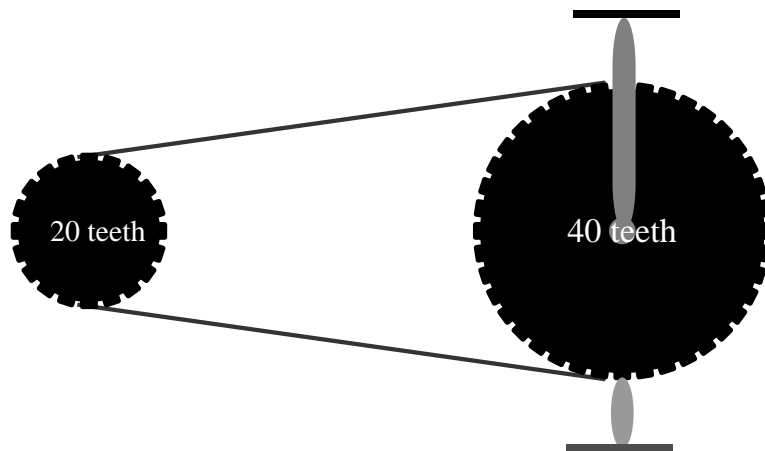
Front sprocket teeth

Rear wheel Diameter

Rear sprocket teeth

Rotation Angle

The gear ratio is 0.5 : 1
Meters of development: 4.14942 meters



A Glimpse Under the Hood

```

Manipulate [
  Deploy[ (* the Deploy command makes the output read only *)
    fMultiplier = frontSprocketTeeth/25; (* these are scaled lengths used for the radii of the gears *)
    rMultiplier = rearSprocketTeeth/25;
    (* pedalV1 & pedalV2 are the points for the corners of the rectangle
    It starts at (3,0) and its center is 1.525*fMultiplier units away
    the (sin, cos) vector rotates it in the opposite direction
    the third vector shifts from the center of rotation to the vertices *)
    pedalV1 = {3, 0} + 1.525*fMultiplier*{Sin[θ], Cos[θ]} + {-0.375*fMultiplier, -0.025*fMultiplier};
    pedalV2 = {3, 0} + 1.525*fMultiplier*{Sin[θ], Cos[θ]} - {-0.375*fMultiplier, -0.025*fMultiplier};
    rearPedalV1 = {2.97, 0} - 1.525*fMultiplier*{Sin[θ], Cos[θ]} + {-0.375*fMultiplier,
-0.025*fMultiplier};
    rearPedalV2 = {2.97, 0} - 1.525*fMultiplier*{Sin[θ], Cos[θ]} - {-0.375*fMultiplier,
-0.025*fMultiplier};
    theta = ArcCos[N[Abs[fMultiplier - rMultiplier]/6]]; (* Calculate the angle between the horizontal
and the larger radius *)
    frontPt = {3 - fMultiplier*(N[Cos[theta]]), fMultiplier*(N[Sin[theta]] + .05)}; (* points of tangency
to form the "chain" *)
    bottomFrontPt = {3 - fMultiplier*(N[Cos[theta]]), -fMultiplier*(N[Sin[theta]] + .05)};
    rearPt = {-3 - rMultiplier*(N[Cos[theta]]), rMultiplier*(N[Sin[theta]] + .05)};
    bottomRearPt = {-3 - rMultiplier*(N[Cos[theta]]), -rMultiplier*(N[Sin[theta]] + .05)};
    myGearTooth = Rectangle[{fMultiplier + 2.92, -0.1}, {fMultiplier + 3.08, 0.1}, RoundingRadius →
0.03]; (* fMultiplier scales the gear teeth down as teeth increase *)
    myRearGearTooth = Rectangle[{rMultiplier - 3.08, -0.1}, {rMultiplier - 2.92, 0.1}, RoundingRadius →
0.03];
    (* rMultiplier scales the gear teeth down as teeth increase, RoundingRadius rounds the edges a bit*)
    myCrank = Rotate[Rectangle[{3.15, 0}, {-0.1*fMultiplier + 3.05, 1.5*fMultiplier}, RoundingRadius →
0.55], -θ, {3, 0}];
    myRearCrank = Rotate[Rectangle[{-0.08*fMultiplier + 3, -1.53*fMultiplier}, {0.08*fMultiplier + 3,
-1.03*fMultiplier}, RoundingRadius → 0.4], -θ, {3, 0}];
    centerHub = Disk[{3, 0}, .15];
    myPedal = Rectangle[pedalV1, pedalV2];
    myRearPedal = Rectangle[rearPedalV1, rearPedalV2];
    minimumTeeth = 9; (* these values are the lowest → highest values on rear "cassettes" *)
    maximumTeeth = 34;
    gearRatio = N[rearSprocketTeeth / frontSprocketTeeth];
    metersOfDevelopment = N[.0254*Pi*rearWheelDiameter*frontSprocketTeeth/rearSprocketTeeth];
    Graphics[{
      Disk[{3, 0}, fMultiplier], (* makes a solid disk centered at (3,0) with radius fMultiplier *)
      {GrayLevel[.2], Thick, Line[{rearPt, frontPt}]}, (* these are the lines for the top & bottom chain
*)
      {GrayLevel[.2], Thick, Line[{bottomRearPt, bottomFrontPt}]},
      Table[Rotate[myGearTooth, 2 π k/frontSprocketTeeth - 2* θ /fMultiplier, {3, 0}], {k, 0,
frontSprocketTeeth - 1}],
      (* rotates the rectangle 2 π/12 radians about the center (1,0) to form front sprocket teeth *)
      {Gray, myCrank}, (* make the crank *)
      {GrayLevel[.6], myRearCrank},
      {Gray, centerHub},(* make a "bolt head" at the center of the hub *)
    ]
  ]
]

```

```

myPedal,(* make the pedal *)
{GrayLevel[.3], myRearPedal},
Disk[{-3, 0}, rMultiplier], (* add the rear sprocket *)
Table[Rotate[myRearGearTooth, 2  $\pi$  m/rearSprocketTeeth - 2* $\theta$  /rMultiplier, {-3, 0}], {m, 0,
rearSprocketTeeth - 1}],
(* rotates the rectangle 2  $\pi$ /12 radians about the center (-5,0) to form rear sprocket teeth *)
Text[Style["The gear ratio is " <> ToString[gearRatio] <> " : 1", FontSize → 20, FontColor → Blue,
Bold], {0.5, 4}],
Text[Style["Meters of development: " <> ToString[metersOfDevelopment] <> " meters ", FontSize → 20,
FontColor → Red, Bold], {0.5, 3.5}],
Text[Style[ToString[rearSprocketTeeth] <> " teeth", FontSize → 14, FontColor → White], {-2.9, 0}],
Text[Style[ToString[frontSprocketTeeth] <> " teeth", FontSize → 16, FontColor → White], {3, 0}]
},
ImageSize → {600, 500}, PlotRange → {{-5, 7}, {-4, 5}}>(* end Graphics *)
],(* end Deploy *)
Text[Style["
Bicycle Gear Analysis", FontSize → 22, Bold]],
Grid[{
{
Grid[{
{
Control[{{frontSprocketTeeth, 30, Style["Front sprocket teeth", Bold, FontSize → 14]}, {30, 40,
51}, SetterBar}], "
",
Control[{{rearWheelDiameter, 26, Style["Rear wheel Diameter", Bold, FontSize → 14], FontSize →
14}, {20, 24, 26, 27, 29}, PopupMenu]}
},
},
Alignment → {Left, Right}
]
},
{
Grid[{
{
Control[{{rearSprocketTeeth, Floor[(minimumTeeth + maximumTeeth)/2], Style["Rear sprocket
teeth", Bold, FontSize → 14]}, minimumTeeth, maximumTeeth, 1, Appearance → "Labeled", ImageSize →
Tiny}], "
",
Control[{{ $\theta$ , 0, Style["Rotation Angle", Bold, FontSize → 14]}, 0, 2 Pi, Appearance → "Labeled",
ImageSize → Tiny, AnimationRate → .01*fMultiplier*rMultiplier]}
}
}
]
},
},
Alignment → Left
]
]
]

```

Resources and References

Mathematica Demonstrations –<http://demonstrations.wolfram.com/>

Bicycle Gear

Accompanying Worksheet

Thank You!

Special Thanks to Everyone who made the RET@ND Program Possible

