

Theorem:

$$0.999\bar{9} = 1$$

Proof:

Let $x = 0.999\bar{9}$, where the bar over the 9 means that the 9s repeat forever to the right.

$$\begin{array}{r} 10x = 9.999\bar{9} \\ -x = -0.999\bar{9} \\ \hline 9x = 9 \end{array}$$

So $x = 1$.

Quod Erat Demonstrandum!

Theorem:

$$\bar{9}999.0 = -1$$

Proof:

Let $x = \bar{9}999.0$, where the bar over the 9 means that the 9s repeat forever to the left.

$$\begin{array}{r} 0.1x = \bar{9}999.9 \\ -x = -\bar{9}999.0 \\ \hline -0.9x = 0.9 \end{array}$$

So $x = -1$.

Quod Erat Demonstrandum!