# Queue Tips!

## Modeling Queue Theories
## in the Marian High School Cafeteria

Marian High school consists of two floors.
The cafeteria is on the first floor.

There are students going through the line over a period of about 25 minutes.
There are three check-out lines, two regular and one a la carte, but a student may choose any line.
A student may buy the standard lunch, a standard lunch with extra items, or a purely a la carte selection.
Students pay by punching in their lunch account number. If the account is overdrawn, there is a delay.

The amount of time a student takes getting through the check-out is the sum of three things:
A. Greeting time - personal interface with the cashier
B. Assessment time - arriving at the total
C. Payment time - debiting the lunch account (or not)

Half the classrooms (and half the student lockers) are on each floor.
Seniors and freshmen have their lockers on the first floor, sophomores and juniors on the second.

Students leave their Before-Lunch-Class (BLC), drop off their books at their lockers, and then head to the cafeteria. This results in three waves of students coming to the cafeteria.

1. Seniors and freshmen with 1st floor BLC have only to visit their lockers (already on the first floor) and go the cafeteria (also on the first floor). Locker-Cafe
2. Next arriving are seniors and freshmen with 2nd floor BLC. They go down the stairs, visit lockers, and go to cafeteria (Stair-Locker-Café). Sophomores and juniors with 2nd floor BLC take about a similar amount of time, only their task order is Locker-Stair-Cafe.
3. Overall, the last to arrive are sophomores and juniors with a 1st floor BLC. They must go upstairs, put books in locker, return down the stairs, and only then go to the cafeteria. Stair-Locker-Stair-Cafe.

This means that the class level of those in line ahead of you depends on when you arrive. Different class levels have different line behaviors.

<u>Seniors</u>
Seniors will be slower in the greeting function if they are in Mrs. Hurley's line. (See below)
Seniors are likelier to slow assessment by buying extra items.

Seniors are the fastest at payment, since they know their account numbers best and are least likely to be overdrawn.

<u>Juniors</u>
Average at greeting
Average at assessment
Slower at payment since they are likeliest to overdraw their account.

<u>Sophomores</u>
Average at all three

<u>Freshman</u>
Freshmen are the fastest at greeting since the cashiers know few of them and they are shyer.
Freshman will be fastest at assessment since they are most likely to get the standard lunch.
Freshmen will be slowest at payment because they are least familiar with their account number.

Which cashier services a line also affects its speed.

<u>Mrs. Rolland</u>
She runs the a la carte line, so her assessment time is always the longest.

<u>Mrs. Hurley</u>
She has a son who is a senior and so is long in greeting her son's friends and former girlfriends.

<u>Mrs. Zansi</u>
She keeps the cafeteria's books and so is quicker at handling payment problems.

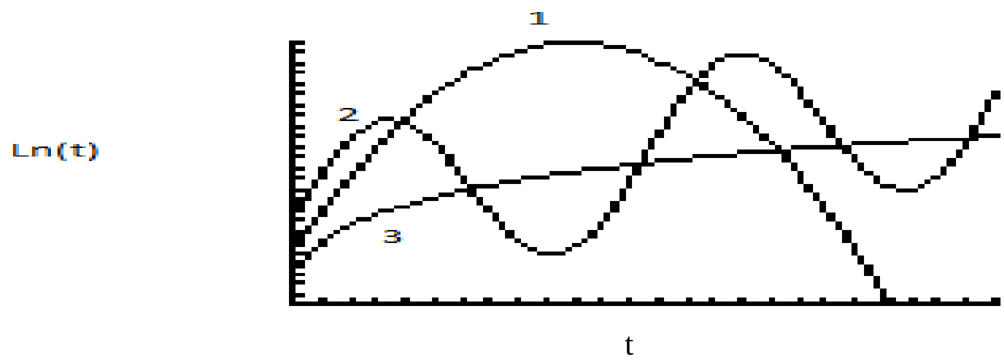
Examples of application.

Avoid Mrs. Hurley's line if there would be a lot of seniors ahead of you.
If you must wait behind freshmen, do so in Mrs. Zansi's line since she will expedite their payment problems.

Let's suppose we could model each line as a function of t (the time you enter the line measured in minutes since the beginning of lunch).

If we graphed them simultaneously, we could observe which line had the shortest wait time at any time t.

Example




If you arrive early, go to line 3.
If you come in towards the end, use line 1.
In the middle of lunch, try line 2.

**Constructing functions.**

If there are n students ahead of you in line, line time can be calculated as follows, where a student k has a greeting time of $g_k$, an assessment time of $a_k$, and a payment time of $p_k$.

$$
\begin{aligned}
T_u &= (g_1 + a_1 + p_1) + (g_2 + a_2 + p_2) + (g_3 + a_3 + p_3) + \ldots + (g_n + a_n + p_n) \\
&= (g_1 + g_2 + g_3 + \ldots + g_n) + (a_1 + a_2 + a_3 + \ldots + a_n) + (p_1 + p_2 + p_3 + \ldots + p_n) \\
&= \sum_{i=1}^{n} g_i + \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} p_i
\end{aligned}
$$

Individual greeting, assessment, and payment times vary with the both the class level of the student and the characteristics of the cashier. (These are listed above.)

Using a computer to simulate arrival time by class, we approximated the percent of seniors in a line with $N_{sr} = 0.000007t^6 - 0.0004t^5 + 0.0087t^4 - 0.0352t^3 - 0.6785t^2 + 4.1379t + 45.464$.
Since freshmen arrive in the same pattern as seniors, $N_{fr} = N_{sr}$.
Juniors and sophomores split what remains, so each is $(100 - 2 \cdot N_{sr})/2$.

Calling this key number, $N_{sr}$, S for simplicity, we find that

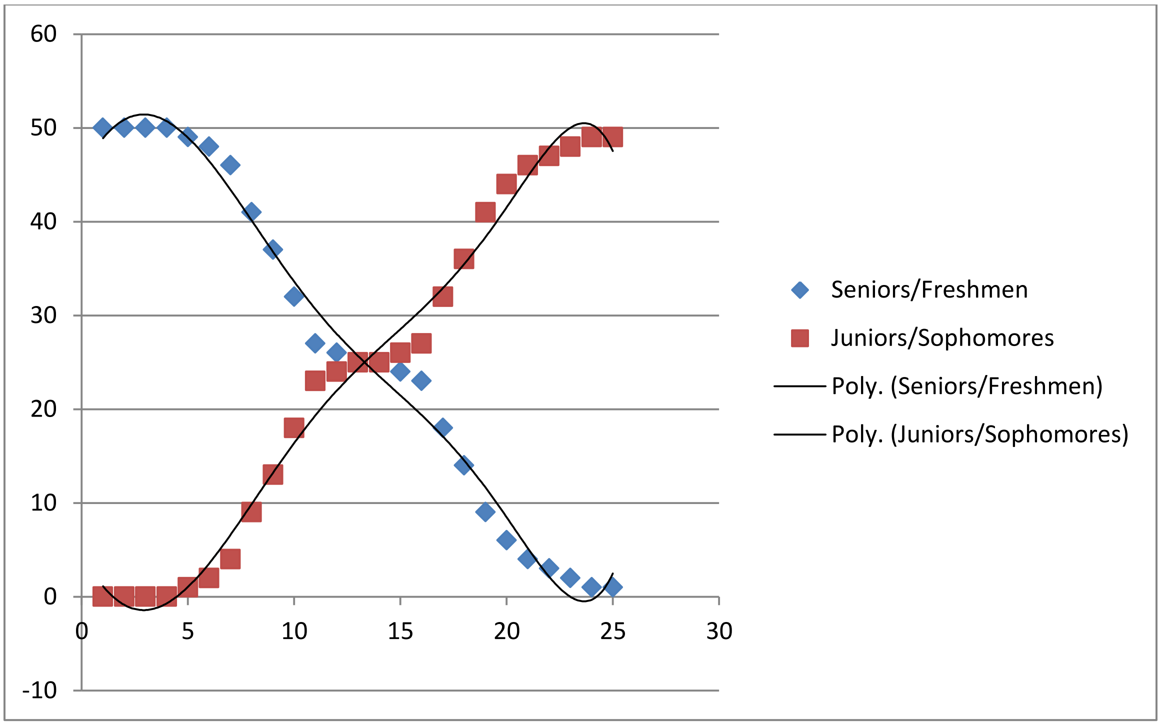
$N_{sr} = S$

$N_{fr} = S$

$N_{jr} = (100-2S)/2 = 50 - S$

$N_{sp} = (100-2S)/2 = 50 - S$

Here is a representation of a line at any time t.
The red is the percent of the line who are seniors, the blue juniors.
(The plot for freshmen is the same as seniors, and sophomores the same as juniors.)




<u>Greeting times by cashier</u>
(# of Seniors)(Senior greet time) + (# of Juniors)(Junior greet time)
+ (# of Sophomores)(Sophomore greet time) + (# of Freshmen)(Freshman greet time)

Zanzi

$G_Z = S\cdot 2g + (50 - S)\cdot g + (50 - S)\cdot g + S\cdot 0.5g$
$= (0.5S + 100)\cdot g$

Hurley

$G_H = S\cdot 4g + (50 - S)\cdot g + (50 - S)\cdot g + S\cdot 0.5g$
$= (2.5S + 100)\cdot g$

Rolland

$G_R = S\cdot 2g + (50 - S)\cdot g + (50 - S)\cdot g + S\cdot 0.5g$
$= (0.5S + 100)\cdot g$

<u>Assessment times by cashier</u>
(# of Seniors)(Senior assess time) + (# of Juniors)(Junior assess time)
+ (# of Sophomores)(Sophomore assess time) + (# of Freshmen)(Freshman assess time)

Zanzi

$A_Z = S\cdot 3a + (50 - S)\cdot 2a + (50 - S)\cdot a + S\cdot a$
$= (S + 150)\cdot a$

Hurley

$A_H = S\cdot 3a + (50 - S)\cdot 2a + (50 - S)\cdot a + S\cdot a$
$= (S + 150)\cdot a$

Rolland

$A_R = S\cdot 3a + (50 - S)\cdot 3a + (50 - S)\cdot 3a + S\cdot 3a$
$= 300a$

<u>Payment times by cashier</u>

(# of Seniors)(Senior pay time) + (# of Juniors)(Junior pay time)
+ (# of Sophomores)(Sophomore pay time) + (# of Freshmen)(Freshman pay time)

Zanzi

$P_Z = 0.8\cdot(S\cdot 0.5p + (50 - S)\cdot 4p + (50 - S)\cdot p + S\cdot 2p)$
$= 0.8(-2.5S + 250)\cdot p$
$= (-2.0S + 200)\cdot p$

Hurley

$P_H = S\cdot 0.5p + (50 - S)\cdot 4p + (50 - S)\cdot p + S\cdot 2p$
$= (-2.5S + 250)\cdot p$

Rolland

$P_R = S\cdot 0.5p + (50 – S)\cdot 4p + (50 – S)\cdot p + S\cdot 2p$
$= (-2.5S + 250)\cdot p$

Time functions by cashier

$T_Z = G_Z + A_Z + P_Z$
$= (0.5S + 100)\cdot g + (S + 150)\cdot a + (-2.0S + 200)\cdot p$
$= 100g + 150a + 200p + (0.5g + a - 2.0p)S$

$T_H = G_H + A_H + P_H$
$= (2.5S + 100)\cdot g + (S + 150)\cdot a + (-2.5S + 250)\cdot p$
$= 100g + 150a + 250p + (2.5g + a – 2.5p)S$

$T_R = G_R + A_R + P_R$
$= (0.5S + 100)\cdot g + 300a + (-2.5S + 250)\cdot p$
$= 100g + 300a + 250p + (0.5g - 2.5p)S$

As mentioned earlier, $S = 0.000007t^6 - 0.0004t^5 + 0.0087t^4 - 0.0352t^3 - 0.6785t^2 + 4.1379t + 45.464$

Using as median approximations, g = 0.0617, a = 0.05, and p = 0.0625,

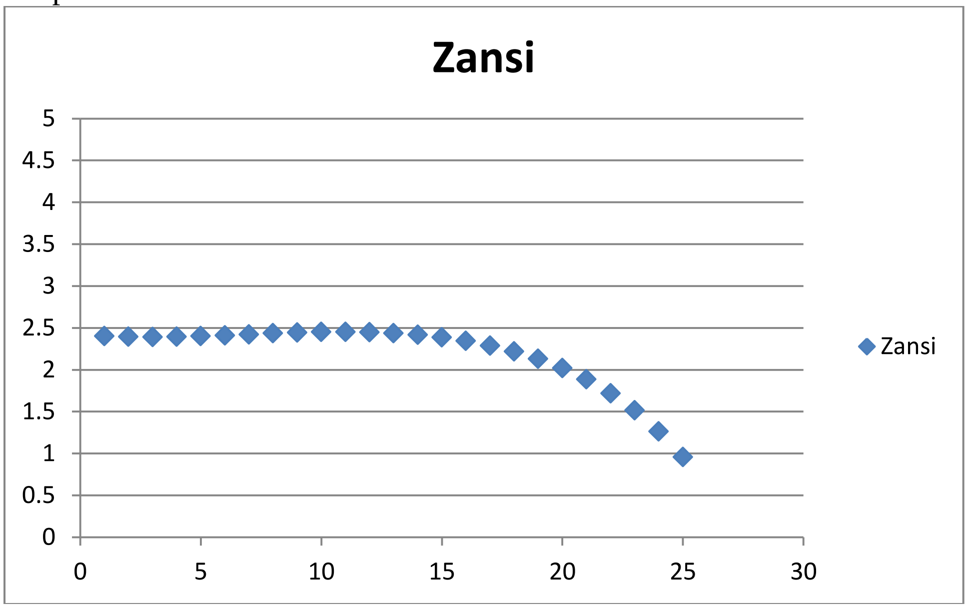
Here are the wait times for each line (measured in minutes).

$$
\begin{aligned}
T_Z &= 100g + 150a + 200p + (0.5g + a - 2.0p)S \\
&= 100(0.0617) + 150(0.05) + 200(0.0625) + (0.5\cdot 0.0617 + 0.05 - 2.0\cdot 0.0625)S \\
&= 6.17 + 7.5 + 12.5 - 0.04415S \\
&= 26.17 - 0.04415(0.000007t^6 - 0.0004t^5 + 0.0087t^4 - 0.0352t^3 - 0.6785t^2 + 4.1379t + 45.464)
\end{aligned}
$$

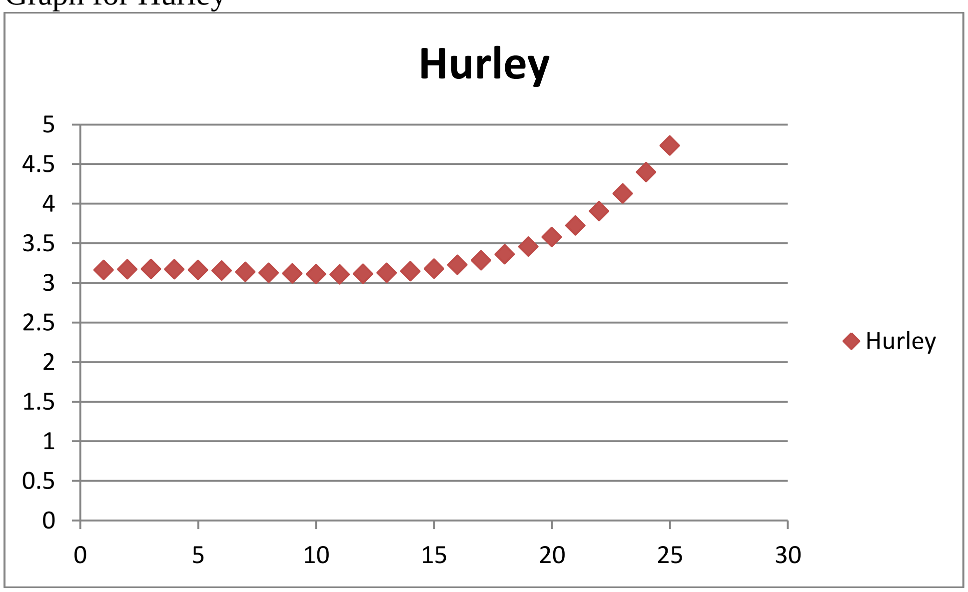
$$
\begin{aligned}
T_H &= 100g + 150a + 250p + (2.5g + a - 2.5p)S \\
&= 100(0.0617) + 150(0.05) + 250(0.0625) + (2.5\cdot 0.0617 + 0.05 - 2.5\cdot 0.0625)S \\
&= 6.17 + 7.5 + 15.625 + .048S \\
&= 29.295 + 0.048(0.000007t^6 - 0.0004t^5 + 0.0087t^4 - 0.0352t^3 - 0.6785t^2 + 4.1379t + 45.464)
\end{aligned}
$$

$$
\begin{aligned}
T_R &= 100g + 300a + 250p + (0.5g - 2.5p)S \\
&= 100(0.0617) + 300(0.05) + 250(0.0625) + (0.5\cdot 0.0617 - 2.5\cdot 0.0625)S \\
&= 6.17 + 15 + 15.625 - 0.0629S \\
&= 36.775 - 0.1254(0.000007t^6 - 0.0004t^5 + 0.0087t^4 - 0.0352t^3 - 0.6785t^2 + 4.1379t + 45.464)
\end{aligned}
$$

Graph for Zansi




Graph for Hurley



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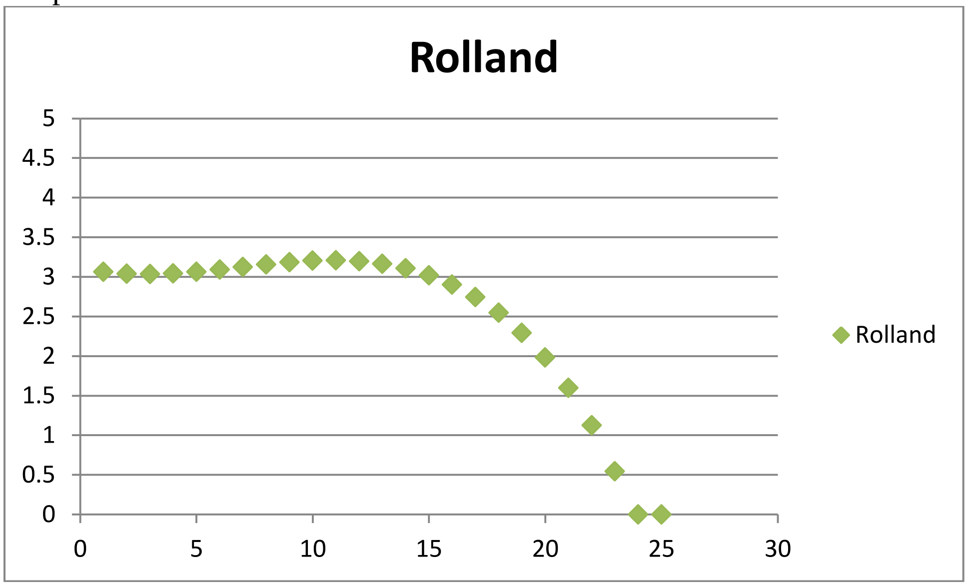
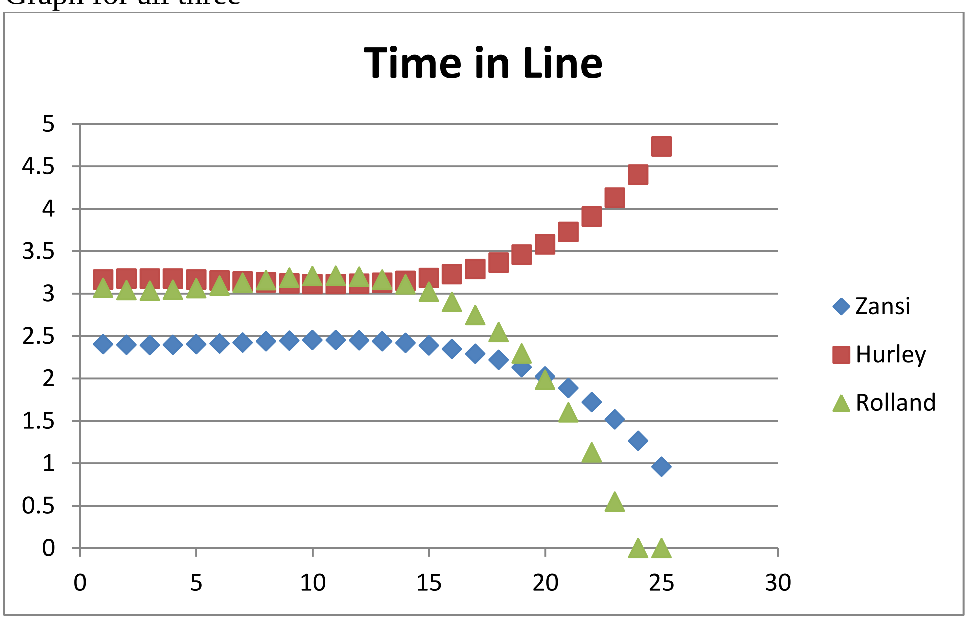
Graph for Rolland




Graph for all three




Conclusions:

Using the created model, it was found that in order to reduce the wait in line, one should go to Zansi for the first 20 minutes. Afterwards, Rolland's line becomes the fastest. In addition, if Zansi's line is unavailable, one should approach Rolland's line for the first 8 minutes, Hurley's from 8-14 minutes, and back to Rolland's for the rest of lunch.